

The values for 1903 to 1915 are based on four stations and those for 1927 to 1952 are based on five. I found that if Kitab is not used the values remain nearly the same. The values of x_1 and y_1 for 1957 were obtained by taking the mean values of these coordinates, as given by Cecchini, for the interval 1954.40 to 1959.20, which contains four Chandler periods. The position of P_1 is plotted in Fig. 10.

An analysis of the data in Table 4 shows that the motion of the mean pole may be represented analytically as the resultant of two

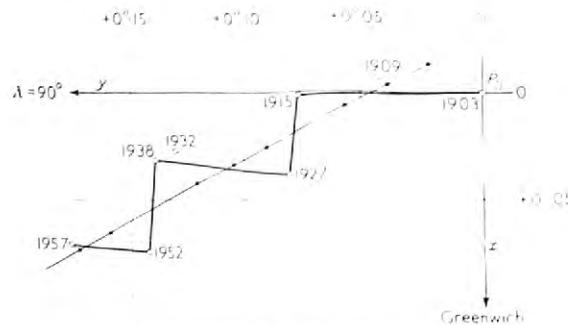


Fig. 10. Position of Mean Pole, P_1 , referred to new system, 1900-95.
Circles are observed positions; P_p indicated by small dots.

motions, a progressive linear motion, P_p , and a periodic libration, P_l . The equations for the coordinates which I consider to be most probable follow. The progressive, or secular motion is

$$\begin{aligned}x_p &= -0''\cdot018 + 0''\cdot0016 (t - 1900) \\y_p &= +0''\cdot014 + 0''\cdot0028 (t - 1900)\end{aligned}\quad (2)$$

The motion in libration is

$$\begin{aligned}x_l &= \pm 0''\cdot012 [\cos 15^\circ (t - 1902)] \\y_l &= -0''\cdot019 [\cos 15^\circ (t - 1902)]\end{aligned}\quad (3)$$

The secular motion is $0''\cdot0032$ yr along the meridian 60° W. The libration takes place along the meridian 122° W, or 58° E, in a period of 24 years. Its coefficient is $0''\cdot022$. The angle between the meridians along which the motions occur is 62° .

Let

$$\begin{aligned}x_2 &= x_1 - x_p & y_2 &= y_1 - y_p \\x_3 &= x_1 - x_l & y_3 &= y_1 - y_l \\x_4 &= x_1 - (x_p + x_l) & y_4 &= y_1 - (y_p + y_l)\end{aligned}$$

Let P_2 , P_3 and P_4 be points which have coordinates (x_2, y_2) ,

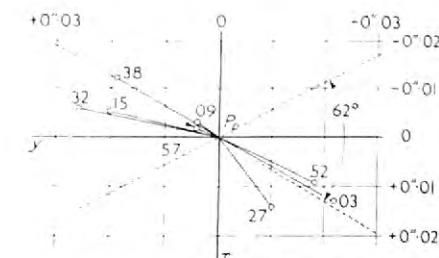


Fig. 11. Libration of Mean Pole.

(x_3, y_3) and (x_4, y_4) , respectively. P_2 , plotted in Fig. 11, is the position of the mean pole with the secular motion removed. P_3 ,

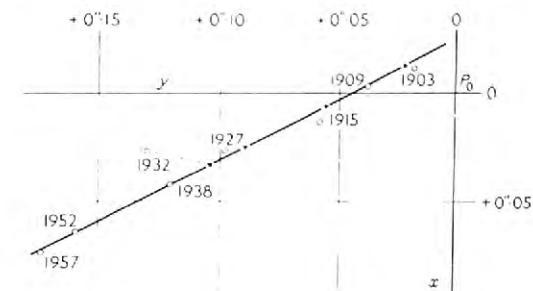


Fig. 12. Secular motion of Mean Pole. Circles represent P_3 ; dots, P_p .

plotted in Fig. 12, is the position of the mean pole with the libration removed. The coordinates of P_1 , P_2 , P_3 and P_4 are given in Table 5.

Table 5. Coordinates of P_1 , P_2 , P^* , P_3 , P_4 , and P_5 . Unit: sec = $0''\cdot001$

Epoch	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5
1903	0	0	13	22	12	18	13	22	12	18
1909	-7	43	4	39	3	3	4	38	0	1
1915	-1	76	6	56	12	18	5	26	13	38
1927	-39	80	25	90	12	18	14	10	27	98
1932	-27	130	33	104	0	6	6	26	27	130
1938	-31	139	43	120	12	19	12	19	43	120
1952	-74	142	65	160	16	17	9	18	64	159
1957	-71	178	73	174	3	6	2	4	74	173

Fig. 11 shows that the position of the mean pole with respect to the uniformly moving pole P_1 always lies near the meridian 122° W, or 58° E. If the motion of the mean pole was erratic because of changes in programme the points P_2 should lie at random. Since they do not, the presumption is that the motion of the mean pole shown in Fig. 10 is not due to programme changes, but to a real motion of the pole.

It is apparent from Fig. 11 that the mean pole moves from one side to the other of the uniformly moving point P_1 , with one exception, in a period of about 24 years. P_2 for 1932 is anomalous with respect to time along the libration path.

On account of the existence of the secular and libration motions the mean pole moves nearly along the meridian 90° W for 12 years and then almost towards Greenwich for 12 years. This effect is well brought out in Fig. 10. It now becomes clear why the rates and directions of the secular motions listed in Table 4 vary: the results listed are closely in accord with what would be expected from the motion shown in Fig. 10. The motion of the mean pole found by Sekiguchi, it may be noted, is nearly the same as that derived from the data of Cechini for the same epochs.

The probable error of a coordinate x_1 or y_1 is $0''\cdot007$, as derived from all 16 values, and $0''\cdot004$ when y_1 for 1932 is not used. In computing these probable errors allowance was made for the fact that 8 constants were selected in deriving equations 2 and 3. The value of y_4 for 1932 was not used in deriving the second equation of 2. Had it been used, the constant term would be $-0''\cdot017$.

6. Station Latitudes

The station latitudes for any programme of observation may be obtained from the formula

$$\Phi = \phi_i - (x \cos \lambda_i + y \sin \lambda_i + z)$$

where ϕ_i , x , y and z are given their mean values for the programme concerned. Using the data of Tables 3 and 4, we obtain the station latitudes listed in Table 6. Subtracting the mean value for each station we obtain residuals v , given in Table 7, which represent the deviations in position for each programme.

In obtaining the entries of Table 6, there were selected 18

Table 6. Station latitudes

Epoch	M	K	C	G	U
1903	3.602		8.941	13.202	12.096
1909	-609		-937	-218	-483
1915	-599		-939	-200	-496
1927	-595	1.844	-945	-186	-402
1932	-596	1.844	-945	-187	-401
1938	-597	1.880	-931	-213	-413
1952	-600	1.856	-939	-203	-400
Mean	3.600	1.856	8.940	13.201	12.099

constants, namely, x , y and z for epochs 1903 to 1952. Five more constants were selected in forming Table 7. Taking into account

Table 7. Residuals, v . Unit: $0''\cdot001$

Epoch	M	K	C	G	U
1903	2		-1	-1	3
1909	9		3	-17	16
1915	1		1	1	3
1927	5	12	-5	-15	-3
1932	4	12	-5	-14	-2
1938	3	24	-9	-12	-14
1952	6	0	1	-2	-1
Mean (v)	3	12	4	9	6

the selection of these 23 constants, it is found that the probable error of a residual v is $0''\cdot011$. This small quantity could be due entirely to accidental and systematic errors of observation and to changes in observers. The residuals are so small for several of the stations, notably Mizusawa, that an absolute fixity with respect to the crust of the earth is indicated.

It appears, thus, that the variation in latitude observed at the stations of the International Latitude Service can be accounted for completely by the motion of the pole plus errors of observation.

7. Independent Stations

One of the principal reasons for the reluctance to accept the reality of the secular motion of the pole, cited by many investigators, has been that observations for latitude made at Greenwich, Pulkovo and Washington indicated that their latitudes were constant. We re-examine this question.

The change in latitude of a station due to the progressive motion, P_p , and the libration, P_l , is of the form

$$\Delta\phi = z(t - 1900) + \beta \cos 15^\circ (t - 1902)$$

where z and β depend upon the longitude. The coefficients for Washington, Greenwich, and Pulkovo are as follows:

	λ	α	β
Washington	77°	0.0031	0.015
Greenwich	0°	0.0016	0.013
Pulkovo	30°	0.0000	0.020

If the motion of the mean pole derived from the I.L.S. results is correct there should be observed a progressive change in latitude and an oscillation of period 24 years. We examine the results obtained with PZT No. 1 at Washington from October 1915 to May 1955 for these terms.

The apparent progressive change in latitude of the PZT depends upon the proper motions used. The ones initially used, that of the *Preliminary General Catalogue*, indicated a change of $+0''.005/\text{yr}$. In 1934 there was introduced a system of proper motions, denoted 1925(1), which was adjusted so as to make the derived latitude nearly constant. The system 1925(1) does not purport, thus, to furnish the secular change in latitude. Fundamental systems of increasing accuracy became available, however, later.

A study of the latitude of the PZT which I made in 1940 indicated that, based on the system of the *General Catalogue*, the latitude was constant within the errors of observation.³⁷

A later study,^{38,39} made in 1945, indicated that the PZT stars, which are faint, would be more accurately represented by the system *FK3*. On this system the progressive change in latitude was $+0''.005/\text{yr}$. I did not, however, regard this as definite proof that the latitude of Washington was increasing because it is not

possible to determine what systematic error may exist in the proper motions of *FK3* near declination $+39^\circ$.

An analysis of the PZT results from 1915 to 1955 shows that on the system 1925(1) the latitude is increasing by $0''.0008/\text{yr}$. The most recent fundamental catalogue available, which contains 19 PZT stars, is *N30*. This catalogue indicates that the proper motions of 1925(1) require a correction of $-0''.0055 \text{ yr}^{-1}$. Hence, the indicated progressive change in latitude of Washington based on

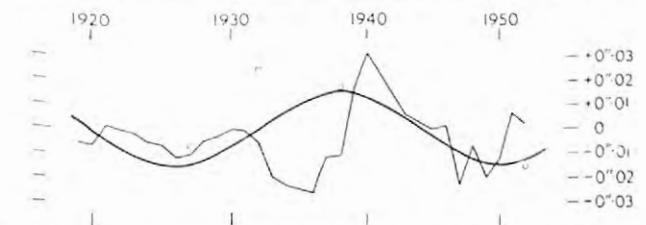


Fig. 13. Six-year running mean latitudes of PZT No. 1, Washington, on system of *N30*, but corrected by $-0''.0063 \text{ yr}^{-1}$. Smooth curve is libration term based on solution for libration of I.L.S. results. Circles are based on observed positions of mean pole of I.L.S.

N30 is $+0''.0063 \text{ yr}^{-1}$. The systematic error in proper motion of *N30* at declination $+39^\circ$ is not known, but it is considered that the rate found is in accordance with that indicated by the I.L.S. results.

To study the periodic term, 6-year running means of the annual mean latitude on *N30* were formed and the term $0''.0063(t - 1900)$ and a constant were subtracted. The resulting values are plotted in Fig. 13. There is also plotted the theoretical variation. The agreement, while not strong, is reasonably good. The circles plotted show the variation to be expected on the basis of the observed positions of the pole given in Table 4.

The coefficient β is small for Greenwich and vanishes for Pulkovo. Hence, the fact that progressive changes in latitude have not been observed at these observatories is in accordance with the I.L.S. results.

8. Variations in Longitude

The study of shifts of continents and the motion of the pole from determinations of longitude is more difficult than from determinations of latitude because time is involved. Also, whereas the

latitude of a station can be determined independently, only differences in longitude for pairs of stations may be determined.

Formerly, longitudes were determined with much less precision than latitudes. Precise instruments, such as the P.Z.T. and the impersonal astrolabe, were not introduced into general use until about 1957, in connection with the International Geophysical Year. About this time, also, the cathode-ray system of monitoring time signals was introduced. Thus, the basis has been laid for obtaining accurate longitude differences which may be compared with other determinations to be made in the future. However, no long series of precise longitude differences exists for the past.

Mme A. Stoyko¹⁰ has utilized such material as is available to study the longitude difference between North America and Europe. From observations for time made from 1924 to 1957 at Washington, Ottawa, and six European observatories she found that the longitude difference of the two continents was decreasing at the rate of $0^{\circ}0012 - 0''018$ yr. This corresponds to a decrease in distance of about 1.3 ft yr.

The material used in the above study was necessarily inhomogeneous, and it seemed desirable to make a new analysis. I selected for study the interval 1934.0–1955.0, during which interval P.Z.T. No. 1 was used for the determination of time at Washington. Observations for time were made at the European observatories with visual instruments.

Each year, N. Stoyko publishes in the *Bulletin Horaire* corrections, K , to the adopted longitude of each observatory which determines time. The yearly values of K were taken out for Washington and the six European observatories, corrected for any changes in adopted longitude between 1934 and 1954, and least square solutions were made for K' , the indicated yearly change in longitude. The values found are as follows for the European observatories:

Observatory	K' sec yr
Greenwich	$-0^{\circ}0009$
Hamburg	$0^{\circ}0000$
Neuchatel	$0^{\circ}0027$
Paris	$0^{\circ}0009$
Potsdam	$0^{\circ}0003$
Uccle	$0^{\circ}0021$
Mean	$0^{\circ}0001$

The value for Washington is $-0^{\circ}0005$ yr, but this requires a correction for proper motion. The visual observations utilize the stars of a fundamental catalogue, but the P.Z.T. stars are too faint to be included therein. Since 1940, for example, *FK3* has been used for time determination. The P.Z.T. proper motions were placed on this system, as best as could be, at that time. They can now be related through *N30*, which allows comparison of 17 P.Z.T. stars in right ascension. A comparison shows that $\mu(N30) = \mu$ (P.Z.T.) $- 0^{\circ}0005$, where μ is the annual proper motion in right ascension. A comparison of *N30* and *FK3* for declinations $+30^{\circ}$ to $+45^{\circ}$ shows that $\mu(FK3) = \mu(N30) - 0^{\circ}0001$. Hence, a correction of $-0^{\circ}0006$ yr is required by the P.Z.T. proper motions to be on *FK3*.

The value of K' for Washington when corrected by this amount becomes $+0^{\circ}0001$ yr. The indicated separation between Washington and the European stations is $0^{\circ}0000$ yr.

It is difficult to calculate a probable error for this determination. The large discordances in the values of K' show that the values are affected by systematic errors. The probable error of a value of K' as derived from the differences from the mean is $0^{\circ}0010$. Other sources of error may exist, but for want of anything better a probable error of $0^{\circ}0010 - 0''015$ yr is adopted.

The motion of the pole must be considered. The variation in longitude due to the polar motion is

$$\Delta\lambda = \tan\phi (\dot{x}\sin\lambda + \dot{y}\cos\lambda) 15$$

where $\Delta\lambda$ is in seconds of time when x and y are in seconds of arc. In obtaining K , N. Stoyko has corrected for the polar motion, using the results of the I.L.S. Stoyko gives the values of x and y which he used, and from these I computed annual means. The progressive changes in x and y from 1934 to 1955 are practically the same as indicated in Fig. 10. The values of K obtained by Stoyko have thus been corrected, in effect, for the total motion of the pole. Hence, within the errors of observation the distance between Washington and the European stations is constant.

IV. DISCUSSION

Studies of the polar motion based on the results of the International Latitude Service indicated at an early date an apparent

wandering of the pole in addition to the 12- and 14-month periodic variations. This wandering was variously interpreted as being due to horizontal displacements of stations, to changes in the direction of the verticals, to asymmetry of observation, and to a real motion of the pole.

In the interval from 1900 to 1959 the mean pole has moved about $0''\cdot2$. It appears unlikely that a change as great as this could be due to asymmetry of observation.

When a term for secular motion is removed from the position of the mean pole, the latter is found to move to and fro, nearly along a straight line. Such an effect could hardly be due to changes in the I.L.S. programme.

On the assumption that the observed variation in latitude is due to the motion of the pole, the probable error of the mean latitude for a station for a 6-year programme is about $0''\cdot01$, which corresponds to 1 ft. Since this could be due to errors of observation, the implication is that the I.L.S. stations are fixed in position with respect to the crust of the earth.

The secular increase in latitude observed with PZT No. 1 at Washington from 1915 to 1955 tends to confirm the value expected from the I.L.S. results.

Similarly, a study of the progressive variations in longitude between the U.S. Naval Observatory and European observatories for the interval 1934–1955 indicates agreement with the I.L.S. secular motion if it is assumed that the observatories are fixed with respect to each other. The confirmation from longitudes, however, has low weight.

The motion of the mean pole from 1900 to 1959, it has been shown, can be expressed as the sum of two motions, a progressive secular motion and a libration of period 24 years. There is no need to assume that local displacements of stations occur to satisfy the observations. When the catalogue of declinations and proper motions of the I.L.S. stars which is being compiled by Melchior is completed we shall be better able to judge the reality of the secular motion found here. In the meantime, it appears more probable that the secular motion is real than that it is due to systematic errors.

To summarize, the polar motion from 1900 to 1959 appears to have three periodic motions

	Period	Coefficient	Nature
(1)	12 mo.	$0''\cdot006$ to $0''\cdot10$	Forced oscillation.
(2)	14 mo.	$0''\cdot08$ to $0''\cdot18$	Free oscillation.
(3)	24 yr	$0''\cdot02$	Libration, cause unknown.

and (4), a secular motion, whose rate is $0''\cdot0032$ yr. The first term is known to be of meteorological origin. The mechanism which sustains the second term is not known.

The third and fourth terms are unexplained, although the melting of ice has been suggested to account for a progressive motion. These terms might also be due to continental uplift. However, no quantitative data are known which would account for the specific motions found.

It is emphasized that the third and fourth terms were found only for the interval 1900–1959. Since a physical cause for their existence has not been given, these terms cannot safely be extrapolated for great eras of time in the past or the future. The progressive term may itself be a libration whose period is as short as several centuries.

If the secular motion continued at its present rate the movement in 10000 years would be 3000 ft, about 1 km. This motion is too small to account directly for the occurrence of ice ages within the last 50000 years.

Assuming that the secular motion of the pole is real, it is safe to say that its characteristics would not have been determined by now but for the continued operation of the chain of stations of the International Latitude Service. Clearly, further study of the motion requires that the results shall be based on chains of stations which observe the same stars.

The PZT's of the Naval Observatory at Washington and the I.L.S. station at Mizusawa observe 40 stars in common.⁴¹ The establishment of a third PZT near latitude $+39^{\circ}$ would provide an independent chain for studying the secular motion of the pole. However, until such time as some new chain of stations is established the International Latitude Service will continue to provide the principal data for studying the secular motion of the pole.

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THE SECULAR MOTION OF THE POLE

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